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## Examples of stationary BPS solutions in $N = 2$ supergravity theories with $R^2$ -interactions

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**Abstract:** We discuss explicit examples of BPS solutions in four-dimensional  $N = 2$  supergravity with  $R^2$ -interactions. We demonstrate how to construct solutions by iteration. Generically, the presence of higher-curvature interactions leads to non-static spacetime line elements. We comment on the existence of horizons for multi-centered solutions.

### 1 Introduction

This note is a follow-up of a recent study [1] of stationary BPS solutions in four-dimensional  $N = 2$  supergravity theories with higher-curvature interactions and with an arbitrary number of abelian vector multiplets and neutral hypermultiplets. Working within the framework of the superconformal multiplet calculus an analysis of solutions with  $N = 2$  and residual  $N = 1$  supersymmetry was presented. It was established that there exist only two classes of fully supersymmetric vacua, namely Bertotti-Robinson and Minkowski spacetime, and that the corresponding solutions are fully specified by the charges carried by the field configurations. As a consequence of the uniqueness of the horizon geometry the so-called fixed-point behavior [2] is thus established in the presence of  $R^2$ -terms. Furthermore, concise equations that govern stationary BPS solutions were derived. In the absence of  $R^2$ -interactions our conclusions agree with previous results (here we refer in particular to the recent work [3]). In this note we demonstrate that the construction of BPS solutions in  $N = 2$  supergravity with higher-curvature interactions is in general very complicated and show, by studying a few explicit examples, how to proceed iteratively. We address the fact that multi-centered BPS solutions are non-static due to the presence of  $R^2$ -interactions, and present arguments why these configurations may still have regular horizons at the charge centers.

## 2 Residual supersymmetry

For what follows we adopt the definitions and conventions used in [1]. The line element of a stationary solution is parametrized by

$$ds^2 = -e^{2g} (dt + \sigma_m dx^m)^2 + e^{-2g} g_{mn} dx^m dx^n. \quad (1)$$

Imposing residual  $N = 1$  supersymmetry subject to the embedding condition

$$h \epsilon_i = \varepsilon_{ij} \gamma_0 \epsilon^j, \quad (2)$$

where  $h$  is a phase, leads to stringent restrictions. For instance,  $g_{mn}$  needs to be a flat three-dimensional metric and the hypermultiplet scalars must be constant. To transparently present the restrictions of residual supersymmetry on the vector multiplet sector it is advantageous to rescale the vector multiplet scalars and to express the Lagrangian in terms of the scale and  $U(1)$  invariant variables  $Y^I$  ( $I = 0, \dots, n$ ), which are shown to be expressed in terms of harmonic functions  $H^I(\vec{x})$  and  $H_I(\vec{x})$  according to

$$\begin{pmatrix} Y^I - \bar{Y}^I \\ F_I(Y, \Upsilon) - \bar{F}_I(\bar{Y}, \bar{\Upsilon}) \end{pmatrix} = i \begin{pmatrix} H^I \\ H_I \end{pmatrix}. \quad (3)$$

These  $2(n+1)$  equations for the real and imaginary parts of the scalars  $Y^I$  constitute the so-called generalized stabilization equations. In the absence of  $R^2$ -terms these equations were first conjectured in [4]. Here  $F_I(Y, \Upsilon)$  is the partial derivative with respect to  $Y^I$  of a homogeneous holomorphic function  $F(Y, \Upsilon)$ . This function characterizes the couplings of the vector multiplets. The field  $\Upsilon$  appearing in  $F(Y, \Upsilon)$  as an extra holomorphic parameter is the (rescaled) scalar of a chiral background superfield which, upon identification with the square of the Weyl superfield, parametrizes the  $R^2$ -interactions. The harmonic functions are related to the electric and magnetic charges carried by the field configurations. In the case of multiple centers they are superpositions of the harmonic functions associated with the electric ( $q_{AI}$ ) and magnetic ( $p_A^I$ ) charges carried by the centers located at  $\vec{x}_A$ ,

$$H^I(\vec{x}) = h^I + \sum_A \frac{p_A^I}{|\vec{x} - \vec{x}_A|}, \quad H_I(\vec{x}) = h_I + \sum_A \frac{q_{AI}}{|\vec{x} - \vec{x}_A|}, \quad (4)$$

where the  $(h^I, h_I)$  are constants. It is remarkable that apart from (3) the only other equations that must be solved to fully specify the stationary BPS solutions, are the ones for the spacetime line element,

$$\begin{aligned} i \left[ \bar{Y}^I F_I(Y, \Upsilon) - \bar{F}_I(\bar{Y}, \bar{\Upsilon}) Y^I \right] + \frac{1}{2} \chi e^{-2g} &= 128i e^g \nabla^p \left[ \left( \nabla_p e^{-g} \right) (F_{\Upsilon} - \bar{F}_{\Upsilon}) \right] \\ &\quad - 32i e^{4g} (R(\sigma)_p)^2 (F_{\Upsilon} - \bar{F}_{\Upsilon}) \\ &\quad - 64 e^{2g} R(\sigma)_p \nabla^p (F_{\Upsilon} + \bar{F}_{\Upsilon}), \\ H^I \overleftrightarrow{\nabla}_p H_I + \frac{1}{2} \chi R(\sigma)_p &= -128i \nabla^q \left[ \nabla_{[p} (e^{2g} R(\sigma)_{q]} (F_{\Upsilon} - \bar{F}_{\Upsilon})) \right] \\ &\quad - 128 \nabla^q \left[ 2 \nabla_{[p} g \nabla_{q]} (F_{\Upsilon} + \bar{F}_{\Upsilon}) \right], \end{aligned} \quad (5)$$

where  $\Upsilon$  is constrained to take the form

$$\Upsilon = -64 \left( \nabla_p g - \frac{1}{2} i e^{2g} R(\sigma)_p \right)^2. \quad (6)$$

Here  $p, q$  denote three-dimensional tangent space indices and  $R(\sigma)_p$  is the dual of the field strength of  $\sigma$ ,  $R(\sigma)^p = \varepsilon^{pqs} \nabla_q \sigma_s$ . Furthermore,  $F_\Upsilon = \partial_\Upsilon F(Y, \Upsilon)$  and  $\chi$  denotes a negative constant which fixes the scale. All other fields, such as the field strengths of the vector multiplets, are given by explicit formulae in terms of  $g$ ,  $R(\sigma)_p$ , and the moduli fields  $Y^I$ . Solutions to the above equations automatically satisfy all equations of motion.

### 3 Examples of BPS solutions

The equations (3), (5), and (6) that determine stationary BPS solutions can be solved explicitly only in very few cases. For instance, it is often difficult to obtain the solution to the generalized stabilization equations (3) in closed form, even in the absence of  $R^2$ -terms. Furthermore, equations (5) and (6) are coupled and can usually be solved only iteratively. For concreteness, let us consider a first simple example,

$$F(Y, \Upsilon) = -\frac{1}{2} i Y^I \eta_{IJ} Y^J + c \Upsilon, \quad (7)$$

where  $\eta$  is a real symmetric matrix and  $c$  a complex number. In this case it is simple to solve the equations (3),

$$Y^I = \frac{1}{2} (i H^I - \eta^{IJ} H_J), \quad F_I = \frac{1}{2} (i H_I + \eta_{IJ} H^J), \quad (8)$$

where  $\eta_{IJ} \eta^{JK} = \delta_I^K$ . The dependence on the  $R^2$ -background will enter only when solving for the line element, as we will discuss shortly. This is rather the exception than the rule. Consider, for instance, the coupling function which arises in Calabi-Yau threefold compactifications in the large-volume limit,

$$F(Y, \Upsilon) = \frac{D_{ABC} Y^A Y^B Y^C}{Y^0} + d_A \frac{Y^A}{Y^0} \Upsilon, \quad (9)$$

with  $A, B, C = 1, 2, \dots, n$ . We construct solutions to this model satisfying  $H^0 = 0$  so that  $Y^0$  is real. Introducing the matrix  $D_{AB} = D_{ABC} H^C$  and assuming its invertibility  $D_{AB} D^{BC} = \delta_A^C$ , the stabilization equations can be solved to all orders in  $\Upsilon$ ,

$$\begin{aligned} Y^A &= \frac{1}{6} Y^0 D^{AB} \left( H_B + i d_B \frac{\Upsilon - \bar{\Upsilon}}{Y^0} \right) + \frac{1}{2} i H^A, \\ (Y^0)^2 &= \frac{D_{ABC} H^A H^B H^C - \frac{1}{3} (\Upsilon - \bar{\Upsilon})^2 d_A D^{AB} d_B - 2(\Upsilon + \bar{\Upsilon}) d_A H^A}{4 \left( H_0 + \frac{1}{12} H_A D^{AB} H_B \right)}. \end{aligned} \quad (10)$$

For more complicated  $F(Y, \Upsilon)$ , solving (3) can become very involved. It may be possible to cast  $F(Y, \Upsilon)$  into a power series expansion (possibly after an electric/magnetic duality transformation) in which case the generalized stabilization equations (3) can be solved iteratively in powers of  $\Upsilon$ ,

$$Y^I(H, \Upsilon, \bar{\Upsilon}) = \sum_{n,m} Y_{(n,m)}^I(H) \Upsilon^n \bar{\Upsilon}^m. \quad (11)$$

Clearly,  $F_I(Y, \Upsilon)$  and  $F_\Upsilon(Y, \Upsilon)$  will have corresponding expansions in  $\Upsilon$  and  $\bar{\Upsilon}$  once the solutions  $Y^I(H, \Upsilon, \bar{\Upsilon})$  are inserted, and one could in principle, by treating  $\Upsilon$  as a formal expansion parameter, proceed to solve (5) and (6). Since such a procedure is not feasible in practice, the question arises whether it makes sense to solve the equations (5) and (6)

iteratively and to truncate at some suitable order. To address this question we recall that the function  $F(Y, \Upsilon)$  is homogeneous of degree two,

$$F(\lambda Y, \lambda^2 \Upsilon) = \lambda^2 F(Y, \Upsilon). \quad (12)$$

It follows that

$$Y^I F_I(Y, \Upsilon) + 2\Upsilon F_\Upsilon(Y, \Upsilon) = 2F(Y, \Upsilon), \quad (13)$$

and in particular we have  $F_I(\lambda Y, \lambda^2 \Upsilon) = \lambda F_I(Y, \Upsilon)$ . This shows that the equations (3) are invariant under this rescaling if we let the harmonic functions  $H^I$  and  $H_I$  scale with weight one. Therefore the coefficient functions  $Y_{(n,m)}^I(H)$  in the expansion (11) will scale with weight  $1 - 2(n + m)$ , such that every power of  $\Upsilon$  is accompanied by a net amount of two inverse powers of harmonic functions  $H^{-2}$ . The expressions (10) illustrate this feature. In a similar way homogeneity is reflected at the level of the Lagrangian. Therefore, corrections due to  $R^2$ -interactions become subleading whenever  $|\Upsilon| \ll H^2$ . One can pinpoint such a hierarchy when one encounters supersymmetry enhancement in the neighborhood of a charged center. There,  $|\Upsilon|$  has a  $1/r^2$ -fall-off proportional to a charge-independent constant, while the harmonic functions fall off as  $Q/r$ , where  $Q$  is the charge carried by the center. This is the reason why the corrections to the entropy of BPS black holes [5], for instance, are subleading in the limit of large charges. In fact, owing to homogeneity, the entropy will have an expansion of the form  $\mathcal{S} = \pi \sum_{n \geq 0} \mathcal{S}^{(n)} Q^{2-2n}$ , where the coefficients  $\mathcal{S}^{(n)}$  are independent of the charges. According to the arguments presented above, homogeneity implies that the expansion of the line element takes the schematic form

$$e^{-2g} \sim \sum_{n \geq 0} \alpha_{(n)} |\Upsilon|^n H^{2-2n}, \quad R(\sigma) \sim \sum_{n \geq 0} \beta_{(n)} |\Upsilon|^n H^{2-2n}, \quad (14)$$

where  $\alpha_{(n)}$  and  $\beta_{(n)}$  are independent of the harmonic functions. This shows that a truncation at some finite order may be sensible only in situations in which  $\Upsilon$  falls off much stronger than the harmonic functions  $H^2$  when moving away from the centers.

Let us reconsider the holomorphic coupling function (7). In this simple example we can use  $c$  as an expansion parameter, since by homogeneity every power of  $c$  will always be accompanied by two inverse powers of harmonic functions. After solving the generalized stabilization equations (8) the remaining equations (5) reduce to

$$\begin{aligned} (H^I \eta_{IJ} H^J + H_I \eta^{IJ} H_J) + \chi e^{-2g} &= 256 i (c - \bar{c}) \left[ e^g \nabla_p^2 e^{-g} - (\tfrac{1}{2} e^{2g} R(\sigma)_p)^2 \right], \\ 2H^I \overset{\leftrightarrow}{\nabla}_p H_I + \chi R(\sigma)_p &= -256 i (c - \bar{c}) \nabla^q \left[ \nabla_{[p} (e^{2g} R(\sigma)_{q]} \right]. \end{aligned} \quad (15)$$

The case where  $c$  is real corresponds to adding a total derivative term to the action. Above formulae show that the line element stays unaltered in this case, while  $\Upsilon$  is given by

$$\Upsilon = 64 \left[ \frac{(H^I - i\eta^{IJ} H_J) \nabla_p H_I - (H_I + i\eta_{IJ} H^J) \nabla_p H^I}{H^I \eta_{IJ} H^J + H_I \eta^{IJ} H_J} \right]^2. \quad (16)$$

There are other less obvious situations where the dependency on  $c$  drops out of the equations (15). This is the case, for instance, when  $R(\sigma)_p = 0$  and all the harmonic functions are proportional to  $e^{-g}$ ,

$$H^I = a^I e^{-g}, \quad H_I = a_I e^{-g}, \quad (17)$$

where  $(a^I, a_I)$  are constants.

Generically, however, the right-hand sides of (15) do not vanish and one has to rely on an iterative approach. For mutually local charges,  $H^I \overset{\leftrightarrow}{\nabla}_p H_I = 0$ , static solutions with  $R(\sigma)_p = 0$  are possible. This is what we want to investigate in the following. The remaining equation,

$$e^{-2g}(\chi - 256 i(c - \bar{c}) e^{3g} \nabla_p^2 e^{-g}) = - (H^I \eta_{IJ} H^J + H_I \eta^{IJ} H_J), \quad (18)$$

is a non-linear differential equation for  $e^{-g}$ . To zeroth order in  $c$  we find

$$[e^{-2g}]^{(0)} = -\chi^{-1} (H^I \eta_{IJ} H^J + H_I \eta^{IJ} H_J). \quad (19)$$

Making the ansatz  $e^{-g} = \sum_{n \geq 0} [e^{-g}]^{(n)} (256 i(c - \bar{c})/\chi)^n$  the line element is determined iteratively by

$$[e^{-g}]^{(n)} = \frac{1}{2} [e^{2g}]^{(0)} \left( \nabla_p^2 [e^{-g}]^{(n-1)} - \sum'_{i,j,k} [e^{-g}]^{(i)} [e^{-g}]^{(j)} [e^{-g}]^{(k)} \right), \quad (20)$$

where the truncated sum  $\sum'_{i,j,k}$  runs over all  $0 \leq i, j, k < n$  subject to  $i + j + k = n$ . The presence of the overall factor  $[e^{2g}]^{(0)} \sim H^{-2}$  on the right-hand side of (20) indeed induces the expansion indicated by (14).

Let us return to the more complicated example (9). In this case we can use the  $d_A$  as expansion parameters. The exact solution to the generalized stabilization equations for the case  $H^0 = 0$  are given in (10). We find by direct calculation that

$$i[\bar{Y}^I F_I - Y^I \bar{F}_I] = -\frac{D_{ABC} H^A H^B H^C}{Y^0} + d_A H^A \frac{\Upsilon + \bar{\Upsilon}}{Y^0}, \quad (21)$$

and

$$F_\Upsilon - \bar{F}_\Upsilon = \frac{id_A H^A}{Y^0}, \quad F_\Upsilon + \bar{F}_\Upsilon = \frac{1}{3} d_A D^{AB} \left( H_B + id_B \frac{\Upsilon - \bar{\Upsilon}}{Y^0} \right). \quad (22)$$

To zeroth order in  $d_A$  the solutions to (5) are given by

$$\frac{1}{2} \chi e^{-2g} = \frac{D_{ABC} H^A H^B H^C}{[Y^0]^{(0)}} + \mathcal{O}(d_A), \quad \frac{1}{2} \chi R(\sigma)_p = -H^I \overset{\leftrightarrow}{\nabla}_p H_I + \mathcal{O}(d_A), \quad (23)$$

where  $[Y^0]^{(0)}$  is the zeroth order expression for  $Y^0$ ,

$$([Y^0]^{(0)})^2 = \frac{D_{ABC} H^A H^B H^C}{4(H_0 + \frac{1}{12} H_A D^{AB} H_B)}. \quad (24)$$

Keeping track of the terms coming from (22) the expressions for the line element up to second order in  $d_A$  are readily obtained from (5),

$$\begin{aligned} \frac{1}{2} \chi e^{-2g} &= \frac{D_{ABC} H^A H^B H^C}{[Y^0]^{(0)}} - 128 \gamma^{-1} \nabla^p \left[ (\nabla_p \gamma) \frac{d_A H^A}{[Y^0]^{(0)}} \right] + 32 \gamma^{-4} (\rho_p)^2 \frac{d_A H^A}{[Y^0]^{(0)}} \\ &\quad - \frac{64}{3} \gamma^{-2} \rho_p \nabla^p (d_A D^{AB} H_B) + \mathcal{O}(d_A d_B), \\ \frac{1}{2} \chi R(\sigma)_p &= -H^I \overset{\leftrightarrow}{\nabla}_p H_I + \frac{256}{3} \nabla^q \left[ \gamma^{-1} (\nabla_{[p} \gamma) \nabla_{q]} (d_A D^{AB} H_B) \right] \\ &\quad + 128 \nabla^q \nabla_{[p} \left( \gamma^{-2} \rho_{q]} \frac{d_A H^A}{[Y^0]^{(0)}} \right) + \mathcal{O}(d_A d_B), \end{aligned} \quad (25)$$

where  $\gamma^2$  and  $\rho_p$  are abbreviations for the zeroth order results as given in (23) for  $e^{-2g}$  and  $R(\sigma)_p$ , respectively. We see again that the first-order approximation to the line element is cast into the form (14), the first correction from the  $R^2$ -terms being suppressed by one power of  $H^{-2}$ .

## 4 Static versus stationary BPS solutions

As pointed out in [1], higher-curvature interactions induce non-static pieces in the line element even for BPS configurations with mutually local charges,  $H^I \overleftrightarrow{\nabla}_p H_I = 0$ . This effect can be observed in the previously discussed example, but it does not arise when the solution has only one center because the right-hand side of the second equation (5) vanishes due to rotational symmetry. Therefore the question arises whether  $R^2$ -interactions still allow for regular multi-centered black hole solutions. Again, it is difficult to address this question in full generality. Due to (14) we expect  $e^{-2g}$  generically to diverge as  $|\vec{x} - \vec{x}_A|^{-2}$  as one approaches one of the charge centers  $\vec{x}_A$ . When the charges are not mutually local one expects  $R(\sigma)_p$  to behave as  $|\vec{x} - \vec{x}_A|^{-3}$ . For mutually local charges, on the other hand, the singularity of  $R(\sigma)_p$  is, in fact, milder. Inspection of the expressions for the curvature components given in [1] shows that every term involving  $R(\sigma)_p$  is accompanied by a sufficient amount of factors of  $e^g$  such that the effect of the non-static pieces of the line element disappears near the centers. This indicates that multi-centered BPS solutions still possess an  $\text{AdS}_2 \times S^2$  geometry there. The following example illustrates this.

Let us consider a simple model describing pure supergravity with the particular  $R^2$ -interactions given by

$$F(Y, \Upsilon) = -\frac{1}{2} i (Y^0)^2 + b \frac{\Upsilon^2}{(Y^0)^2}. \quad (26)$$

We assume  $b$  to be a real constant. By the same arguments as given in the previous section, we use  $b$  as an expansion parameter. We solve the generalized stabilization equations for the purely electric situation  $H^0 = 0$ ,  $H_0 \equiv H$ . To zeroth order in  $b$  we find

$$e^{-2g} = -\chi^{-1} H^2 + \mathcal{O}(b), \quad R(\sigma)_p = \mathcal{O}(b), \quad \Upsilon = -64 H^{-2} (\nabla_p H)^2 + \mathcal{O}(b). \quad (27)$$

To calculate the next-order corrections to the line element one needs  $F_\Upsilon + \bar{F}_\Upsilon$  to first order in  $b$ ,

$$F_\Upsilon + \bar{F}_\Upsilon = -2^{10} b H^{-4} (\nabla_p H)^2 + \mathcal{O}(b^2). \quad (28)$$

We consider a harmonic function  $H$  with multiple centers located at  $\vec{x}_A$ . For simplicity let us assume that center  $A = 1$  is at  $\vec{x}_1 = 0$  and calculate  $R(\sigma)_p$  around this center. Therefore we expand the harmonic function appearing in above expression in powers of  $|\vec{x}|$ ,

$$\begin{aligned} H &= h + \sum_A \frac{q_A}{|\vec{x} - \vec{x}_A|} = \frac{q_1}{|\vec{x}|} + h + \sum_{A \neq 1} \frac{q_A}{|\vec{x}_A|} + \mathcal{O}(|\vec{x}|), \\ \nabla_p H &= -q_1 \frac{x_p}{|\vec{x}|^3} + \sum_{A \neq 1} q_A \frac{x_{Ap}}{|\vec{x}_A|^3} + \mathcal{O}(|\vec{x}|), \\ \nabla_p \nabla_q H &= q_1 \left( 3 \frac{x_p x_q}{|\vec{x}|^5} - \frac{\delta_{pq}}{|\vec{x}|^3} \right) + \sum_{A \neq 1} q_A \left( 3 \frac{x_{Ap} x_{Aq}}{|\vec{x}_A|^5} - \frac{\delta_{pq}}{|\vec{x}_A|^3} \right) + \mathcal{O}(|\vec{x}|). \end{aligned} \quad (29)$$

Near  $|\vec{x}| \approx 0$  one finds that, to first order in  $b$ ,  $R(\sigma)_p$  is given by

$$R(\sigma)_p = \frac{3 \cdot 2^{19} b}{\chi q_1^3} \frac{\delta_{pq} + \hat{x}_p \hat{x}_q}{|\vec{x}|} \sum_{A \neq 1} q_A \frac{(x_A)^q}{|\vec{x}_A|^3} + \mathcal{O}(b^2), \quad (30)$$

where  $\hat{x}_p$  denote the components of the unit vector. Thus,  $R(\sigma)_p$  exhibits only a  $|\vec{x}|^{-1}$  singularity so that the Riemann curvature near the center  $A = 1$  is not affected. Hence, in this simple example we recover the usual  $\text{AdS}_2 \times S^2$  geometry typical for extremal black holes.

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